We’re comparing two subsets of time series from the M3 monthly data.

1. Yearly Industry series with large forecast error increase (relative to the original forecast error): 41 time series that had a 258% increase in MAE under k-nts+ (k = 3).
   1. Selected Features: *variance, spike, mean, x\_acf1, hurst, max variance shift, kurtosis*
2. Monthly Micro series with small forecast error increase (relative to the original forecast error): 474 time series that had a 14.9% increase in MAE under k-nts+ (k = 3).
   1. Selected Features: variance, mean, spike, max variance shift, max level shift

The features that are selected *might* be predictive of whether our protection methodology will perform well (give a desirable trade-off between privacy and forecast accuracy). Cases where parameters based on auto-correlation are selected (e.g., x\_acf1, hurst) seem to perform poorly whereas cases where only features based on distributions of the points are selected seems to perform better.

Figure 1 contains plots of the original series from each subset. The series with the large forecast error increase are a little “smoother” than the noisy time series that had a small forecast error increase. Also of note is that the time series values are less dense in the set of time series on the left, i.e., there may be fewer suitable neighbors for swapping. The series on the left have lower spectral entropy values and are easier to forecast.

***Figure 1****: plots of the original time series that had a large increase in forecast error (left) and the time series that had a small increase in forecast error (right).*

A graph showing different colored lines

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We assess the privacy/utility tradeoff using additive noise with noise parameter . For each time period of each series, a protected value is created by adding random noise proportional to the standard deviation of the original time series:

Where

And is the standard deviation of the th time series.

Figure 2 shows the mean absolute error for SES and DES forecasting models applied to the original time series and to protected series with values of ranging from 0.05 to 3.0. The time series shown on the left in Figure 1 have a low MAE for small values of which quickly increases as increases. The series on the right in Figure 1, had a much more gradual increase in forecast error as increased (these series had a higher original forecast error to begin with). This supports the notion that protecting series that have poor forecast accuracy to begin with will not seriously degrade forecast accuracy.

***Figure 2****: mean absolute error across SES and DES forecasting models and all series as a function of the Additive Noise parameter s. Note that corresponds to the original data.*

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The average spectral entropy of the protected data sets exhibits a similar pattern to the MAE values. The series on the left start with a relatively low average spectral entropy (indicating good forecastability) but exhibit a rapid increase in the average spectral entropy as increases. The series on the right have very high average spectral entropy to begin with. In other words, we don’t make the forecast results much worse by swapping values between the series on the right.

***Figure 3****: average spectral entropy across all series as a function of additive noise parameter s. Note that corresponds to the original data.*

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***Figure 4****: Feature distributions from the original and k-nTS+ versions of the series with the large forecast error increase.*

A group of boxes with numbers and symbols

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Figure 4 plots some of the feature distributions from the original and k-nTS+ versions of the time series that had a large increase in forecast error. Some of the feature distributions are well maintained, such as the mean and max variance shift. These features measure overall location and spread of the time series values. Features that depend on temporal relationships *between* points are poorly maintained. For example, the feature e\_acf1, which is the autocorrelation of the error component of the decomposed time series, shows moderate autocorrelation in the original series that is destroyed in the protected series.

***Figure 5****: Feature distributions from the original and k-nTS+ versions of the series with the small forecast error increase.*

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Description automatically generated with medium confidence

Figure 5 plots some of the feature distributions from the original and k-nTS+ versions of the time series that had a small increase in forecast error. Most of the feature distributions are very well maintained. Overall, the features that depend on the temporal relationships between between time series points are much weaker in these time series. For example, the error component of the decomposed time series exhibits very little autocorrelation.

Our method appears capable of consistently maintaining features that depend on the overall distribution of the time series points (e.g., mean, variance, skewness, kurtosis). We made this observation when assessing the two M3 subsets in this document as well as two M4 subsets in the other document. These results suggest that the features that RFE chooses as most important can predict the success (or usefulness) of our method on a given data set.